Drawing down an Annuity

Lecture 5

Robb T. Koether

Hampden-Sydney College

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Outline

Annuity Formula (Drawing Down)

- The formula for the amount to withdraw each period:

$$M = P\left(\frac{r/k}{1-\left(1+\frac{r}{k}\right)^{-kt}}\right),$$

where M is the amount withdrawn per period, P is the amount in the annuity when the withdrawals begin, r is the annual interest rate, and t is the number of years.

Annuity Formula (Drawing Down)

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  where M is the amount withdrawn per period, P is the amount in the annuity when the withdrawals begin, r is the annual interest rate, and t is the number of years.
- If the withdrawals are annual, then $k = 1$ and the formula becomes

$$M = P\left(\frac{r}{1-(1+r)^{-t}}\right).$$

Example

Example (Five Withdrawals)

- Suppose that a person has accumulated $10,000 and that it is earning 10% interest per year.
- How much can he withdraw each year for 5 years?

Example

Example (Three Withdrawals)

The amount withdrawn is

$$M = \frac{Pr}{1 - (1 + r)^{-t}}$$

Example

Example (Three Withdrawals)

The amount withdrawn is

$$M = \frac{Pr}{1-(1+r)^{-t}} = \frac{(10000)(.10)}{1-(1.10)^{-5}}$$

Example

Example (Three Withdrawals)

The amount withdrawn is

$$\begin{aligned} M &= \frac{Pr}{1-(1+r)^{-t}} \\ &= \frac{(10000)(.10)}{1-(1.10)^{-5}} \\ &= 2637.97. \end{aligned}$$

Example

Example (Three Withdrawals)

Year	Starting Balance	Interest	Total	Withdrawal	Ending Balance
1	10,000.00	1,000.00	11,000.00	2,637.97	8,362.03

Example

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Example

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3	6,560.26	656.03	7,216.29	2,637.97	4,578.32

Example

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2	8,362.03	836.20	9,198.23	2,637.97	6,560.26
3	6,560.26	656.03	7,216.29	2,637.97	4,578.32
4	4,578.32	457.83	5,036.15	2,637.97	2,398.18

Example

Example (Three Withdrawals)

Year	Starting Balance	Interest	Total	Withdrawal	Ending Balance
1	10,000.00	1,000.00	11,000.00	2,637.97	8,362.03
2	8,362.03	836.20	9,198.23	2,637.97	6,560.26
3	6,560.26	656.03	7,216.29	2,637.97	4,578.32
4	4,578.32	457.83	5,036.15	2,637.97	2,398.18
5	2,398.18	239.82	2,638.00	2,637.97	0.03

Outline

Example

Example (10-Year Example)

- Suppose we invest $200.00 each month at 9% for 18 years for a college savings account.
- Then we withdraw from the account a fixed amount (to be determined) each year for the next 4 years (tuition payments).

Example

Example (Building up the Annuity)

The future value is of the annuity is

$$F = \frac{P((1 + \frac{r}{12})^{12t} - 1)}{r/12}$$

Example

Example (Building up the Annuity)

The future value is of the annuity is

$$F = \frac{P((1+\frac{r}{12})^{12t}-1)}{r/12}$$
$$= \frac{200((1.0075)^{216}-1)}{0.0075}$$

Example

Example (Building up the Annuity)

The future value is of the annuity is

$$\begin{aligned} F &= \frac{P((1+\frac{r}{12})^{12t}-1)}{r/12} \\ &= \frac{200((1.0075)^{216}-1)}{0.0075} \\ &= \$107,270.33 \end{aligned}$$

Example

Example (Drawing down the Annuity)

- Now we begin making withdrawals over the next 4 years.
- How much can we withdraw each year?

Example

Example (Drawing down the Annuity)

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$$M = \frac{Pr}{1 - (1 + r)^{-t}}$$

Example

Example (Drawing down the Annuity)

- Now we begin making withdrawals over the next 4 years.
- How much can we withdraw each year?

$$M = \frac{Pr}{1-(1+r)^{-t}}$$
$$= \frac{(107270.33)(0.09)}{1-(1.09)^{-4}}$$

Example

Example (Drawing down the Annuity)

- Now we begin making withdrawals over the next 4 years.
- How much can we withdraw each year?

$$\begin{aligned} M &= \frac{Pr}{1-(1+r)^{-t}} \\ &= \frac{(107270.33)(0.09)}{1-(1.09)^{-4}} \\ &= \$33,100.99 \end{aligned}$$

Example

Example (Drawing down the Annuity)

- What if the interest rate were 10%?

Example

Example (Drawing down the Annuity)

- What if the interest rate were 10%?
- What if the interest rate were 12%?

Outline

Another Example

Example

- That same person says, “But I think I’ll need $50,000 each year for tuition and I’m afraid that I will earn only 6% on the average.”
- How much should the person invest each month?

Example

Example (Drawing down the Annuity)

- We have to work the problem “backwards.”
- What must be the value of the annuity in order to withdraw $50,000 each year for 4 years?

Example

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- We have to work the problem “backwards.”
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$$M = \frac{Pr}{1 - (1 + r)^{-t}}$$

Example

Example (Drawing down the Annuity)

- We have to work the problem “backwards.”
- What must be the value of the annuity in order to withdraw \$50,000 each year for 4 years?

$$M = \frac{Pr}{1-(1+r)^{-t}}$$

$$50000 = \frac{P(0.06)}{1-(1.06)^{-4}}$$

Example

Example (Drawing down the Annuity)

- We have to work the problem “backwards.”
- What must be the value of the annuity in order to withdraw \$50,000 each year for 4 years?

$$M = \frac{Pr}{1-(1+r)^{-t}}$$

$$50000 = \frac{P(0.06)}{1-(1.06)^{-4}}$$

$$= P(0.28859149)$$

Example

Example (Drawing down the Annuity)

- We have to work the problem “backwards.”
- What must be the value of the annuity in order to withdraw \$50,000 each year for 4 years?

$$
\begin{aligned}
M &= \frac{Pr}{1-(1+r)^{-t}} \\
50000 &= \frac{P(0.06)}{1-(1.06)^{-4}} \\
&= P(0.28859149) \\
P &= \frac{50000}{0.28859149}
\end{aligned}
$$

Example

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- We have to work the problem “backwards.”
- What must be the value of the annuity in order to withdraw \$50,000 each year for 4 years?

$$
\begin{aligned}
M &= \frac{Pr}{1-(1+r)^{-t}} \\
50000 &= \frac{P(0.06)}{1-(1.06)^{-4}} \\
&= P(0.28859149) \\
P &= \frac{50000}{0.28859149} \\
&= \$173,255.28
\end{aligned}
$$

Example

Example (Building up the Annuity)

- Now how much must be invested each month at 6% interest in order to have $173,255.28 after 18 years?

Example

Example (Building up the Annuity)

- Now how much must be invested each month at 6% interest in order to have \$173,255.28 after 18 years?

$$F = \frac{P((1 + \frac{r}{12})^{12t} - 1)}{r/12}$$

Example

Example (Building up the Annuity)

- Now how much must be invested each month at 6% interest in order to have \$173,255.28 after 18 years?

$$F = \frac{P((1+\frac{r}{12})^{12t}-1)}{r/12}$$

$$173255.28 = \frac{P(1.005^{216}-1)}{0.005}$$

Example

Example (Building up the Annuity)

- Now how much must be invested each month at 6% interest in order to have \$173,255.28 after 18 years?

$$F = \frac{P((1+\frac{r}{12})^{12t}-1)}{r/12}$$

$$\begin{aligned} 173255.28 &= \frac{P(1.005^{216}-1)}{0.005} \\ &= P(387.35319) \end{aligned}$$

Example

Example (Building up the Annuity)

- Now how much must be invested each month at 6% interest in order to have \$173,255.28 after 18 years?

$$F = \frac{P((1+\frac{r}{12})^{12t}-1)}{r/12}$$

$$173255.28 = \frac{P(1.005^{216}-1)}{0.005}$$

$$= P(387.35319)$$

$$P = \frac{173255.28}{387.35319}$$

Example

Example (Building up the Annuity)

- Now how much must be invested each month at 6% interest in order to have \$173,255.28 after 18 years?

$$F = \frac{P((1+\frac{r}{12})^{12t}-1)}{r/12}$$

$$\begin{aligned} 173255.28 &= \frac{P(1.005^{216}-1)}{0.005} \\ &= P(387.35319) \\ P &= \frac{173255.28}{387.35319} \\ &= \$447.28 \end{aligned}$$

Outline

Assignment

Assignment

- Annuity worksheet: 4, 5, 6.